Subsidizing the R&D Expenditures for a Monopoly Firm: Advice for NIST

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ABSTRACT. The Advanced Technology Program (ATP) of the National Institute for Standards and Technology (NIST) subsidizes the R&D expenditure of large single firms at a maximum rate of 40%. The theoretical analysis herein of a monopoly innovator suggests that this subsidy rate is about socially optimal when spillovers to other industries are small and only incremental R&D expenditure is subsidized. The optimal subsidy when these two conditions are not met is also discussed. (O38)

I. Introduction

The United States government, like the governments of many other countries, subsidizes some of the Research and Development expenditure of firms. Since the early 1990s, many of these subsidies have been administered through the Advanced Technology Program (ATP) of the National Institute for Standards and Technology (NIST). Although the study is somewhat dated now, Connie K.N. Chang (1998), Supervisory Economist at NIST-ATP listed five analogues to ATP, for Canada, E.U. Finland, Japan, and the U.K. Subsidy rates were at most 50% for all programs except Canada, where the maximum was 20-25%, university projects in the E.U. which could be 100% subsidized, and Japan where a subsidy rate was not given. At a subsequent "International Panel on Funding R&D Projects" meeting of the ATP Advisory Committee 11 March 2003, Japan's subsidy rates were given as up to 50% for private companies, the E.U.'s remained the same, while Canada's were 30-50%. (See the ATP site http://www.atp.nist.gov/atp/application.htm.)

The goal of this paper is to investigate the optimal subsidy rate for a monopoly innovator which introduces a new product into an established "commoditized" market, i.e., one served by competitive marginal cost-pricing firms. Although we focus on an American firm and the U.S.'s ATP program, the basic analysis of the paper could be applied to the subsidy programs of other countries as well. It is assumed that the

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government (i.e., ATP) expects the new product to be developed and brought to market by a firm, and that the firm will choose the profit maximizing price. The maximum subsidy given by ATP for a large single company is 40%.

The present paper argues that a subsidy rate of one third of the firm's total R&D maximizes social surplus unless there are benefits to subsidizing R&D which are not captured by the researching firm and its customers. Perhaps not surprisingly, an *ad hoc* analysis which considers the latter possibility suggests that when spillovers are positive the subsidy rate should be higher; negative spillovers suggest a lower subsidy rate. Finally, the socially optimal subsidy rate is also affected by the design of the funding program—specifically, whether all of a firm's R&D is subsidized or only the "incremental R&D," that is, R&D in excess of what would be done by the firm if the subsidy rate were zero. The socially optimal subsidy of such incremental R&D is explored by simulation.

Perhaps the most important feature of the model of the present paper concerns the structure of R&D expenditure and quality. In the quality literature, R&D is often a costly activity that produces something called "quality" from which potential buyers derive utility or for which they have some willingness to pay. Formulating quality this abstractly has the advantage of being useful in a wide variety of settings. But measuring what it is precisely that consumers see as quality is quite difficult; in practice a proxy for quality is usually used. (See Berry and Waldfogel (2003) for a typical model with two quite different empirical applications and several proxies for quality).

Instead of thinking of quality as simply an abstract aspect of a product, the present paper returns to an earlier notion suggested by Matthews and Moore (1987). They measure quality as the probability that a product functions ("succeeds") or one minus the probability that it fails. When quality is measured in this way, the costs of failure must explicitly be taken into account. Suppose product failure imposes a cost on customers, for example, the costs associated with returning the product. Customers do not know which exact unit of the product will fail, but they know the probability of a failure and the costs imposed on them of a failure, and they adjust their reservation price for the product accordingly. Reservation prices are distributed uniformly, and the marginal cost of producing the product is constant per unit. In brief, the innovating firm can undertake R&D expenditures which reduce the probability of product failure. One of the most important consequences of treating quality this way is that it is a "metric"—that is, something that can be objectively measured either by the firm's quality control division, or by an outside agency.

Consumer theory in this paper falls in the general line of the Sutton and Shaked literature (e.g., Shaked and Sutton (1987) and Sutton (1991)) in the sense that a distributional foundation for customer behavior is assumed. The customer behavior of the present paper is similar to Highfill, Polley, and Scott (2004) except that they largely ignore issues of quality. Distributional demand is common in the quality literature as well. The present paper draws particularly on Herguera and Lutz (2003), Chaudhuri (2000), and Herguera, Kujal, and Petrakis (2000) who in turn are motivated by Motta (1993) for the notions of consumer surplus and social surplus. The present paper also follows Herguera and Lutz (2003), Chaudhuri (2000), and Herguera, Kujal, and Petrakis (2000) for important supply side assumptions: the cost of quality is quadratic and depends only on the level of quality, i.e., R&D expenditure is independent of the number of units produced. The differences between these three references and the present paper are that they consider a duopoly, and do not allow for an existing "commoditized" product. The latter aspect of the model is adapted from De Fraja (1996). The cost of product failure in the present paper is similar to a cost of use function for the customer (as in Jung, 2004).

Much of the literature relating R&D and subsidies considers duopoly and international competitiveness issues. Examples include Zhou, Spencer, and Vertinsky (2002), Park (2001), Jinji (2003), Toshimitsu (2003), Okamoto (1998), and Herguera and Lutz (2003). Studies of the effect of a subsidy for a single firm are less common. In Bagwell (1991) the subsidy is motivated by a desire to increase exports. Jung (2004) considers a subsidy mechanism for the case when an innovator provides two qualities for two types of buyers while Siebert (2003) considers the effect of a subsidy on a high quality producer as compared to a low quality producer. Hsu and Schwartz (2003) have a tantalizing working paper which argues that pull subsidies are more effective than push subsidies (i.e., subsidizing cost) when the research process itself is uncertain. And finally, in a general equilibrium growth setting, Zeng (2001) argues that an innovation subsidy speeds up growth while an imitation subsidy does the opposite. In sum, although a number of important issues with regard to the subsidization of R&D expenditure have been examined, the question of an optimal subsidy in the very simple case of a monopoly innovator in a single country seems to have been overlooked.

II. Customer Behavior

Suppose potential customers can choose between an existing product on the market and a new product which is being supplied by a monopoly firm—the "innovating firm." Suppose customers have identical reservation prices for the existing product, which is supplied by a competitive industry so that its price is equal to its marginal cost of production. A consumer who buys the existing product gets a consumer surplus of CS_0, which is constant over all customers who buy the existing product.

The innovating firm chooses the quality of its product by choosing an appropriate level of research and development expenditure. While customers have identical reservation prices for the existing product, their reservation prices for the innovator's product are in general not identical. Suppose reservation prices (denoted v) for the new product (at its highest possible level of quality) are uniformly distributed on the interval $(0,V)$. Suppose when the product "fails" the customer returns it and exchanges it for a new one; if the second product fails the customer returns and exchanges it again, and so forth. (A product "fails" when its performance does not meet the wants and needs of the consumer so that he or she returns it.) The cost imposed on the customer by the failure is captured by the parameter K.

The expected number of exchanges, T, is

$$T = \frac{P_F}{1 - P_F} \tag{1}$$

recalling

$$\frac{P_F}{1-P_F} = \frac{1}{1-P_F} - 1 = P_F + P_F^2 + P_F^3 + \cdots .$$

Notice that for a perfect good $T = 0$; a *lower* quality product is thus associated with a higher value of T and vice versa. Defining P as the usual concept of price, i.e., the price paid by the customer when the product is originally purchased, the "full quality price" of the product is $P + K\,T$ which includes the expected cost of a product failure.

Customers whose reservation price minus the full quality price is greater than the consumer surplus of the existing product, CS_0 , will buy the innovator's product. Those whose reservation price is less that the full quality price of the innovator's product plus CS_0 will buy the existing product. Customers are indifferent between the two products when

$$v - (P + K\,T) = CS_0 \,. \tag{2}$$

Please see Figure 1; reservation prices are on the horizontal axis, the population distribution function $1 \,/\, V$ is on the vertical axis.

Figure 1. Customer Behavior

Figure 1 shows the proportion of customers who buy the existing product and the proportion who buy the innovator's product. The quantity demanded of the innovator's product is the proportion of potential customers who buy it times the number of potential buyers, denoted N. Quantity demanded, Q, is thus

$$Q = N\int_{P+KT+CS_0}^{V} \frac{1}{V} dv = N\left(1 - \frac{P + KT + CS_0}{V}\right).$$

The indirect demand function for the innovator is

$$P = V - CS_0 - KT - \frac{V}{N}Q. \tag{3}$$

It will be useful to note that consumer surplus is

$$\begin{aligned} CS &= N\int_0^{P+KT+CS_0} \frac{CS_0}{V}dv + N\int_{P+KT+CS_0}^{V} \frac{(v-(P+KT))}{V}dv \\ &= \frac{N}{2V}\left((P+KT+CS_0)^2 - 2(P+KT)V + V^2\right) \\ &= \frac{N}{2V}\left((V-\frac{V}{N}Q)^2 - 2(V-\frac{V}{N}Q-CS_0)V + V^2\right) \\ &= N\cdot CS_0 + \frac{1}{2}\frac{V}{N}Q^2. \end{aligned} \tag{4}$$

III. Research and Development and Quality

The present section explains the relationship between the research and development expenditure of the innovator, *RDE*, and the quality of the innovator's product, the expected return rate, *T*. In general, the higher the expenditure on R&D, the better the quality of the product produced. This relationship is captured by a cost of quality function

$RDE = Z(T)$ with $Z'(T) < 0$ $Z''(T) > 0$,

which is assumed to be independent of the number of units produced, or equivalently

$$T = T(RDE) \text{ with } T = Z^{-1} \text{ and } T'(RDE) < 0,\ T''(RDE) > 0. \tag{5}$$

Although the main results of the paper concerning the optimal subsidy will be proven using the general function (5), the intuition of the paper may be aided by considering a specific functional form. Suppose

$$RDE = z(T_0 - T)^2 \quad \text{or} \quad T = T_0 - \sqrt{\frac{RDE}{z}} \tag{6}$$

where $T_0 > 0$ is the "technology endowment," i.e., level of quality available without R&D expenditure, and $z > 0$ is a parameter. For the illustrations and examples that follow (6) will be used.

Figure 2. Measures of Quality as a Function of R&D Expenditure

The expected return rate, T, is the consumer's measure of quality, but, of course, it is derived from the probability that the product fails or succeeds, so the latter are measures of quality as well. Figure 2 illustrates the relationship between R&D expenditure and all of these measures. As more is spent on research and development, the probability of a success increases, the probability of a failure falls, as does the expected number of returns. Notice that when $RDE = 0$, the probability of a failure is not 100%.

IV. The Innovating Firm's Optimization Problem

The innovating firm is assumed to be a simple profit maximizer. Production costs are constant per unit (and independent of the level of quality). The relationship between R&D expenditure and quality is given in the previous section. The government may subsidize the research and development of the technology that produces quality; the subsidy rate, if any, is taken as given by the firm. The government's decision-making regarding the subsidy is considered below.

It is a straightforward exercise to show that the same optimal solution is achieved whether the firm uses price or quantity as one choice variable, and one of the quality variables (P_F, P_S, T, RDE) as the other choice variable. But in order to aid the intuition, it will be convenient to use (Q, *RDE*) as the choice variables.

Similarly, more that one definition of a subsidy "rate" is possible. A given dollar subsidy could be thought of in relation to all R&D expenditure of the firm, or in relation to only the "incremental" R&D, that is, the increase in R&D spending by the firm elicited by the subsidy as compared to the R&D expenditure when the subsidy is zero. For example, suppose a firm's *RDE* would be \$10 with no subsidy. Now suppose that a subsidy of \$10 is matched by the firm with an *RDE* of \$10. (Total *RDE* would be \$30—\$20 paid by the firm and \$10 by the government.) The subsidy as a proportion of all R&D spending would be $S \equiv Subsidy\ /\ Total\ RDE$ = \$10 / \$30 = 1/3. The subsidy rate as a proportion of the firm's incremental *RDE* would be *Subsidy / Incremental RDE* = \$10 / \$20 = 1/2. The formal analysis of the paper is simplified by using the first subsidy definition, but of course, for a given set of parameters, the first definition implies the second and vice-versa. Both concepts will be discussed in the examples and simulations.

Finally, a one or two stage decision making process for the innovator can be assumed without changing the outcome, but the exposition of the paper assumes a two-stage maximization problem (which is solved by backward induction). Specifically, the innovator decides quality in the first stage and then chooses the appropriate quantity in a "take-to-market" stage. The two stages allow for the quality to be determined (and double-checked if need be) after the first stage but before any subsidy is paid and the product is taken to market.

The profit function for the innovator is

$$\begin{aligned} \Pi &= (P - mc)Q - RDE(1 - S) \\ &= (V - CS_0 - K\,T(RDE) - \frac{V}{N}Q - mc)Q - RDE(1 - S) \end{aligned} \tag{7}$$

where *mc* is the (constant) marginal cost of production and *S* is the subsidy rate defined above. The second line of (7) has substituted from the indirect demand function (3), and the abstract quality function (5). In the backward induction process, the firm chooses *Q* to maximize profits for a given level of R&D expenditure:

$$\Pi_Q = (V - CS_0 - K\,T(RDE) - mc) - 2\frac{V}{N}Q = 0 \tag{8}$$

Solving this for Q as a function of RDE

$$Q(RDE) = \frac{V - CS_0 - KT(RDE) - mc}{2\dfrac{V}{N}}. \tag{9}$$

From the indirect demand function (3) and (8), writing P as a function of RDE

$$P(RDE) = V - CS_0 - KT(RDE) - \frac{V}{N}Q(RDE). \tag{10}$$

With these the profit function (7) can be written as only a function of RDE

$$\Pi = (P(RDE) - mc)Q(RDE) - RDE(1 - S) \tag{11}$$

and the profit maximizing condition

$$\Pi_{RDE} = (P(RDE) - mc)\frac{dQ(RDE)}{dRDE} + \frac{dP(RDE)}{dRDE} - (1 - S) = 0. \tag{12}$$

The left-hand side of (11) is the marginal variable profit with respect to RDE,

$$MVP_{RDE} \equiv (P(RDE) - mc)\frac{dQ(RDE)}{dRDE} + \frac{dP(RDE)}{dRDE},$$

while the right-hand side is the proportion of the R&D expenditure that is paid by the innovating firm. For the example of quadratic costs, notice that (12) can be written

$$\frac{KN}{4Vz}\left(K + \sqrt{\frac{z}{RDE}}(V - CS_0 - KT_0 - mc)\right) - (1 - S) = 0. \tag{12'}$$

For the second order condition

$$\Pi_{RDE,RDE} = -\frac{KN(V - CS_0 - KT_0 - mc)}{8V\sqrt{z}}RDE^{-3/2} < 0.$$

The interpretation of the optimal R&D expenditure condition is given in Figure 3. The marginal variable profit with respect to *RDE* is downward sloping, so the firm-optimal R&D expenditure increases when the subsidy rate increases. Once the firm-optimal *RDE* is determined from (12') or in Figure 3, the firm-optimal quality can be determined from (6) or Figure 2.

Figure 3. Firm-Optimal R&D Expenditure

The question remains of what the optimal subsidy is from society's point of view. To see this, it will be convenient to define social surplus, write it as a function of *RDE*, and compare it to the innovator's profits as a function of *RDE*. The innovator's profits (11) can be conveniently written

$$\Pi(RDE) = \frac{V}{N} Q(RDE)^2 - (1 - S)RDE \tag{13}$$

or variable profits

$$VP(RDE) = \frac{V}{N} Q(RDE)^2 \tag{14}$$

See Figure 4. Assume for a moment that the subsidy rate is zero. For very low levels of R&D expenditure, an increase in expenditure increases variable profits more than the increase in R&D expenditure itself, and profits go up. For higher levels of R&D expenditure, profits top out, and for even higher R&D expenditure, increases in R&D cause profits to decline.

Figure 4. Profit and Social Surplus as a Function of R&D Expenditure

Define social surplus (given that the firm markets the product) as consumer surplus (4) plus variable profits (14) less the expenditure on research and development.

$$\begin{aligned} SS &= N \cdot CS_0 + \frac{1}{2}\frac{V}{N}Q(RDE)^2 + \frac{V}{N}Q(RDE)^2 - RDE \\ &= N \cdot CS_0 + \frac{3}{2}\frac{V}{N}Q(RDE)^2 - RDE. \end{aligned} \tag{15}$$

See Figure 4 again. The innovating firm maximizes profits by choosing an *RDE* which is less than the expenditure which maximizes social surplus. Thus, in general, in the absence of a government subsidy for R&D expenditure, too little research and development is done, and too little quality provided.

V. The Socially Optimal Subsidy

The goal of the present section is to find the subsidy rate that maximizes social surplus. Recall the reduced form of the social surplus function:

$$SS = N \cdot CS_0 + \frac{3}{2}\frac{V}{N}Q(RDE)^2 - RDE. \tag{15}$$

Notice that in (15) equation (5) rather than (6) is assumed—that is, an abstract quality function is assumed rather than a specific functional form. To find the socially-optimal subsidy, think of *RDE* as a function of the subsidy rate *S* (for example, as in (12)), and differentiate (15)

$$\frac{dSS}{dS} = 3\frac{V}{N}Q\frac{dQ}{dRDE}\frac{dRDE}{dS} - \frac{dRDE}{dS} = 0. \tag{16}$$

Now recalling (13), the profit maximizing condition can be rewritten

$$2\frac{V}{N}Q\frac{dQ}{dRDE} = (1 - S). \tag{12''}$$

(To show that (12") is the same as (12), use (7) and the indirect demand function.)

Substituting from (12'') into (16)

$$\frac{dSS}{dS} = \frac{dRDE}{dS}\left(3\frac{(1-S)}{2} - 1\right) = 0. \tag{16'}$$

or $1 - S = \frac{2}{3}$, and $S = \frac{1}{3}$.

An example of the effect of the subsidy is shown in Table 1 (the parameters are given in Table1b).

The subsidy elicits a large increase in R&D spending by the firm, with the concomitant improvement in the quality measures. Price, sales, and profits all increase, but notice that the full quality price, which includes the expected cost of a product failure, declines. The overall effect is to increase social surplus by about twenty percent. Finally, notice that the subsidy as a proportion of the firm's incremental *RDE* is about 42%—a point that will be discussed further below.

TABLE 1a–Numerical Example, Primary Results

Variable	$S = 0$	$S = 1/3$	% Change
Quantity Sold	10.69	15.61	46.0%
Market Share	11.9%	17.3%	46.0%
Prob. Success	0.83	0.96	15.3%
Expected Return Rate	0.200	0.041	-79.7%
Price	59.51	63.89	7.4%
Full Quality Price	70.49	66.11	-6.2%
Profit	62.31	91.01	46.0%
Var. Profit	101.61	216.72	113.3%
R&D Expenditure	39.29	188.57	379.9%
RDE from Firm (1-S) RDE	39.29	125.72	220.0%
RDE from Government S RDE	0.00	62.86	--
Consumer Surplus	51.70	109.26	111.3%
Social Surplus	114.02	137.41	20.5%
Govt's Share of Incremental R&D	0.0%	42.1%	--

TABLE 1b–Parameters, Numerical Example

Parameters	Value
V	80
CS_0	0.01
K	55
N	90
mc	50
z	2200

To see how representative the results of Table 1 are, a Monte Carlo simulation (20,000 iterations) was done that picked parameter values from the following distributions: $V \sim U(0,100)$, $N \sim U(0,100)$, $mc \sim U(0,100)$, $K \sim U(0,V)$, $z \sim U(0,10000)$, $T_0 \sim U(0,1)$,$CS_0 \sim U(0,1)$. In order to have economically interesting results, only solutions where the increase in social surplus was 10% or more (between the cases where $S = 0$ and $S = 1/3$) were considered, and the firm's R&D (without a subsidy) had to be between 2% and 20% of its revenue. The results are reported in Table 2.

TABLE 2a–Results: Basic Simulation

Variable	*S* = 0 (Avg)	*S* = 1/3 (Avg)	% Change (Avg)	% Change (Std.Dev.)
Quantity	7.96	11.63	53.8%	34.4%
Market Share	12.2%	17.7%	53.8%	34.4%
Prob. Success	0.72	0.86	20.2%	9.8%
Expected Return Rate	0.42	0.18	-60.3%	22.6%
Price	53.53	57.92	8.6%	4.1%
Full Quality Price	69.07	64.67	-6.5%	2.7%
Profit	59.20	82.64	53.8%	34.4%
Var. Profit	91.88	185.67	148.4%	147.3%
R&D Expenditure	32.68	154.55	458.9%	331.4%
RDE from Firm (1-S) RDE	32.68	103.03	272.6%	221.0%
RDE from Government S RDE	--	51.52	--	--
Consumer Surplus	70.20	117.09	73.8%	55.2%
Social Surplus	129.40	148.21	16.3%	2.7%
Govt's Share of Incremental R&D	--	42.1	--	--

The qualitative results of the averages in Table 2 are quite close to the example in Table 1, except that the increase in social surplus is about 16% rather than 20%. The firm's R&D expenditure increased substantially on average, from \$32.68 to \$103.03 (i.e., 221%), so the subsidy appears to have the desired result—it induces a considerable increase in R&D by the innovator. The standard deviation of the firm's change in *RDE* is also large, but since the minimum percentage increase was 132.7% (not shown in the table) the firm's R&D always at least doubled, usually tripled, and sometimes increased more than that. The government's share of R&D expenditure was \$51.52 on average, so dividing this figure by the average increment in R&D (154.55-32.68 =

$121.75) gives the government's share of incremental R&D as 42.3%, on average. Thus the average results for the simulation are very close to the result in the example in Table 1.

TABLE 2b–Parameter Statistics: Basic Simulation

Parameter	Average	Std. Dev.	Maximum	Minimum
V	78.98	17.07	99.98	14.32
CS_0	0.37	0.27	0.99	0.00
K	40.41	13.21	75.94	5.87
N	65.45	23.88	100.00	4.75
mc	43.61	14.57	80.84	4.72
z	934.43	603.64	3871.55	26.18
Sample Size 649				

This section has shown that the socially optimal subsidy $S \equiv$ *Subsidy / Total RDE* is one third, while the simulation suggests that the alternation notion of a subsidy rate *Subsidy / Incremental RDE* is just over 40%—the maximum subsidy provided large single firms by NIST. But notice that other countries sometimes have higher subsidy rates, which raises the question of how large the subsidy can get before it becomes counter productive. To begin thinking about this question, consider social surplus and profits as a function of the subsidy rate *S*, as in Figure 5.

Figure 5. Profit and Social Surplus as a Function of the Subsidy Rate

The innovator always benefits from an increase in the subsidy rate—but society does not. In fact, there is an upper bound on the subsidy, labeled $S_{\max}$, that gives the same level of social surplus as if there were no subsidy at all. In the case of quadratic costs,

$$S_{\max} = \frac{2(K^2N - 4V_z)}{3K^2N - 16V_z}.$$

For the simulation whose results are reported in Table 2, the average $S_{\max}$ is 43%. Since ATP's maximum subsidy rate is 40%, the implication is that, on average, ATP's subsidies increase social surplus (as compared to the zero subsidy case) even if they don't maximize social surplus. And this is case, even when, as has been assumed so far, all the benefits of the innovator's *RDE* accrue to itself and its customers. Simulation results showing the effect of non-optimal subsidy rates on the innovating firm and the economy are available from the authors. Although not the focus of this paper, various comparative static results are available from the authors as well.

The possibility of "spillovers" is considered next.

VI. Spillovers: An *Ad Hoc* Sharing of Research and Development Costs

The previous discussion of the optimal subsidy assumes that only the innovator and its customers derive benefits from the research and development expenditure of the innovator—and thus they should bear the entire cost of the R&D. Suppose on the one hand, that some of the benefits of the R&D accrue to other parties (other firms and/or consumers) in society, and that these others pay a portion of *RDE*. For example, suppose the innovator and its own customers derive 95% of the benefits of the R&D and thus are expected to pay 95% of the R&D costs (while society at large picks up the remaining 5% of the costs). Denote by $\theta > 0$ the proportion of costs borne by the innovator and its customers (95% in the example).

On the other hand, there may be cases where all the costs of this innovation are not borne by either the innovator or its customers. In this case θ would be greater than one; for example if there are negative externalities amounting to 5% of the R&D expenditure, θ would be 1.05.

With this *ad hoc* assumption the subsidy should be chosen to maximize

$$SS = N \cdot CS_0 + \frac{3}{2}\frac{V}{N}Q(RDE)^2 - \theta\, RDE \tag{17}$$

In this case, the subsidy is the solution to

$$\frac{dSS}{dS} = 3\frac{V}{N}Q\frac{dQ}{dRDE}\frac{dRDE}{dS} - \theta\frac{dRDE}{dS} = 0. \tag{18}$$

Substituting from (12'') into (18)

$$\frac{dSS}{dS} = \frac{dRDE}{dS}\left(3\frac{(1-S}{2} - \theta\right) = 0. \tag{18'}$$

or

$$1 - S = \theta\frac{2}{3}, \text{ and } S = 1 - \frac{2\theta}{3}. \tag{19}$$

The subsidy is positive as long as $\theta < 1.5$.

In the examples given above, when $\theta = .95$, the subsidy would be $S = 11 / 30 \approx .367$. When $\theta = 1.05$, the subsidy would be $S = .3$. If as much as 10% of the benefits to the R&D accrue to other markets so that $\theta = .9$, the subsidy would be $S = \frac{2}{5} = .4$.

Figure 6. *Ad Hoc* Subsidy Rate

To summarize the results of this *ad hoc* calculation see Figure 6. The lower line is $S = 1 - \frac{2\theta}{3}$ from (19) (graphing S on the vertical axis). This is lower bound of the subsidy divided by the incremental R&D because the lower bound on the *RDE* without a subsidy is zero. The rest of the information in Figure 6 is derived from a simulation exactly like that described in the previous section, except that $\theta \sim U(0,1.5)$. The upper frontier of the subsidy per incremental R&D (i.e., "Govt. % of Δ R&D") is convex rather than linear, as is suggested by the upper line shown (intercept .985 and slope .5) and the regression line between the two other lines (intercept 1.01 and slope .57). The highest subsidy per incremental R&D is 89%, when θ is close to zero—the case when positive spillovers are quite large. The lowest subsidy per incremental R&D is 15%, when θ is close to 1.5—the case when negative spillovers are relatively large. For a given θ, the largest range of subsidies per incremental R&D is when θ is .9. As mentioned, when $S = \frac{2}{5} = .4$ the range of subsidies per incremental R&D is from .4 to .54, a range of almost fifteen percentage points. Similar ranges when θ is .5 (the axis shown) are .67 to .75 and when θ is 0 (the case assumed in the previous sections of the paper) are .37 to .48.

Looking at Figure 6 from the other perspective, notice that the range of θ's for which the subsidy per incremental R&D is about .4 goes from $\theta = .93$ to $\theta = 1.08$. Thus as long as spillovers, whether positive or negative, are not greater than 10% of the overall benefits, a subsidy per incremental R&D of about 40% seems reasonable.

VII. Conclusion

Without a doubt the issues surrounding the socially optimal subsidy rate are more complex than can be discussed in this paper, even when, as assumed herein, there is a single innovating firm. (The socially optimal subsidy in the case of more than one firm is a question worthy of investigation in its own right; future work is certainly needed.) Our results suggest that under certain extreme conditions the optimal subsidy can be either zero or one. Nevertheless, our results suggest that ATP's maximum subsidy rate of 40% is reasonable under a number of different scenarios. Our results further suggest that even when 40% is not the socially optimal subsidy rate, on average, social surplus is greater than it would be if the subsidy rate were zero. Finally, although the focus of this paper has been on ATP's 40%, the subsidy rates in other countries (30%-50%) are not unreasonable under the right circumstances as well.

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